

ANALYSIS OF MARKED GRAPHS OF BASIC STOCK CONTROL SYSTEM AND KANBAN CONTROL SYSTEMS USING SIGN INCIDENCE MATRIX

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ABSTRACT:

In this paper we take four marked graphs of one basic stock control system and three kanban control systems and analyze them using sign incidence matrix .

Keywords: Petri Nets, Marked Graphs, Flexible Manufacturing Systems, Basic Stock Control System, Simultaneous Kanban Control System, Independent Kanban Control system, Independent Extended Control system, Sign Incidence Matrix .

INTRODUCTION

A petri net is a powerful modeling formulation in a computer science, system engineering and many others disciplines. A Petri net combines a well-defined mathematical theory with a graphical representation of the dynamic behavior of the system the theoretic aspects of Petri net allows precise modeling and analysis of system behavior while the graphical representation of Petri net enables visualization of the modeled system state change. Because of this Petri net has been used to model various kinds of dynamic event driven system like computer networks, communication system, manufacturing plant etc. Generating Basis Siphons and traps of Petri Nets using the Sign Incidence The analysis of marked graphs using sign Incidence matrix. we take the marked graph model of four different FMS given in and analyze them using the method.

This paper is organized as follows section I contains some basic definitions .section II Contains the algorithm. Section III contain four examples of marked graphs of Flexible manufacturing systems which are analyzed using sign incidence matrices. Section IV contains conclusion and references.

SECTION I

Definition 1.1: A PN is a bipartite graph, where nodes are classified as places and transitions (graphically pictured as circles and bars, respectively), and directed arcs connect only nodes of different type. Places are endowed with integer variables called tokens. More formally, a marked PN is a 5-tuple $N = (P, T, F, W, M_0)$, where P is a finite set of places, T is a finite set of transitions, with $P \cap T = \emptyset, F \subseteq (P \times T) \cup (T \times P)$ is the incidence or flow relation (each element of F corresponds to an arc in the PN), $W : F \rightarrow N \setminus \{0\}$ is the arc weight function, and $M_0 : P \rightarrow N$ is the initial marking (a marking $M : P \rightarrow N$ defines the distribution of tokens in places), where N is the set of natural numbers.

Definition 1.2: Marked graph: A Marked graph is a Petri net in which each place as exactly one input transition and one output transition.

Definition 1.3: Flexible Manufacturing System: A Flexible manufacturing system (FMS) is an integrated computer controlled configuration of machine tools and automated material handling devices that simultaneously process medium sized volumes of a variety of part types. Flexible manufacturing system is a discrete event dynamical system in which the work pieces Of various job classes enter the system asynchronously and are Concurrently, sharing the limited resources , viz., workstations, robots, MHS, buffers and so on.

SECTION II

Definition 2.1: For a Petrinet N with n -transitions and m -places, the sign incidence matrix $A = [a_{ij}]$ is an $n \times m$ matrix whose entry is given as follows.

$$\begin{aligned} a_{ij} &= + && \text{if place } j \text{ is an output place of transition } i. \\ a_{ij} &= - && \text{if it is an input place of transition } i. \\ a_{ij} &= \pm && \text{if it is both input and output places of transition } i \text{ (i.e. transition } i \text{ and place } j \text{ form a self loop)} \\ a_{ij} &= 0 && \text{otherwise.} \end{aligned}$$

Definition 2.2: The addition denoted by \oplus is a commutative binary operation on the set of four element $B = \{+, -, 0, \pm\}$ defined as follows.

$$\begin{aligned} + \oplus - &= \pm \\ x \oplus x &= x, && \text{For every } x \in B \\ \pm \oplus x &= \pm, && \text{For every } x \in B \\ 0 \oplus x &= x, && \text{For every } x \in B \end{aligned}$$

Definition 2.3: A subset of places denoted as S in Petri Net N is called a siphon if $*S \subseteq S^*$, i.e every transition having an output place in S has a input place in S . A nonempty subset of places Q in a Petri Net N is called a trap if $Q^* \subseteq *Q$ i.e every transition having an input place in Q has an output place in Q .

2.4 Enumeration of siphon and trap subsets of places of marked graphs: Here we present an algorithm given in [4] for marked graphs to find all subsets of places which are both siphon and trap. We define a siphon-trap matrix for marked graphs. A relation between sign incidence matrix and siphon-trap matrix for marked graphs is obtained.

Theorem 2.5: A subset of k -places $Z = \{p_1, p_2, \dots, p_k\}$ in a marked graph N is both siphon and trap if and only if the addition of k -column vectors of the sign incidence matrix of N , $A_1 \oplus A_2 \oplus \dots \oplus A_k$ contains either zero entry or \pm entry where A_i denote the column vector corresponding to place P_i , $j = 1, 2, \dots, k$.

Definition 2.6: A $+$ entry is said to be neutralized by adding a $-$ entry to get a \pm entry.

Algorithm 2.7:

Input: Sign incidence matrix A of order $m \times n$.

Step 1 Select A_j , the first column in the sign incidence matrix A , whose corresponding place is denoted as $PLACE_j$. Set recursion level r to 1

Set $V_{jr} = A_j$

Set $PLACE_{j_r} = PLACE_j$

Step 2 If V_{jr} has a \pm entry at i^{th} row then $PLACE_j$ is a self loop with transition t_j .

Step 3 If V_{jr} has a $+$ entry in the k^{th} row find a column in A , which contains a $-$ entry at the k^{th} row.

(a) If no such column in A exists, Go to step 5.

If such A_s exists add it to V_{jr} to obtain $V_{j(r+1)} = V_{jr} \oplus A_s$ containing a \pm entry at k^{th} row. Then $PLACE_{j(r+1)} = PLACE_{j_r} \cup PLACE_s$.

Repeat this step for all possible neutralizing columns A_s . This gives a new set of $V_{j(r+1)}$'s and $PLACE_{j(r+1)}$'s.

Step 4 Increment r by 1. Repeat step 3 until there are no more $+$ entries in each

$V_{jr} = A_1 \oplus A_2 \oplus \dots \oplus A_{jr}$ or no neutralizing column can be defined.

Step 5 Any V_{jr} without $+$ entries and without $-$ entries (i.e., all the entries are either zero or \pm) represents siphon and trap (By theorem). i.e., the places in $PLACE_{j_r}$ form both siphon and trap.

Step 6

Delete A_j

$j=j+1$

Go to step 1.

Output: All sets which are both siphon and trap.

SECTION –III

Example 3.1. Basic Pull Control System

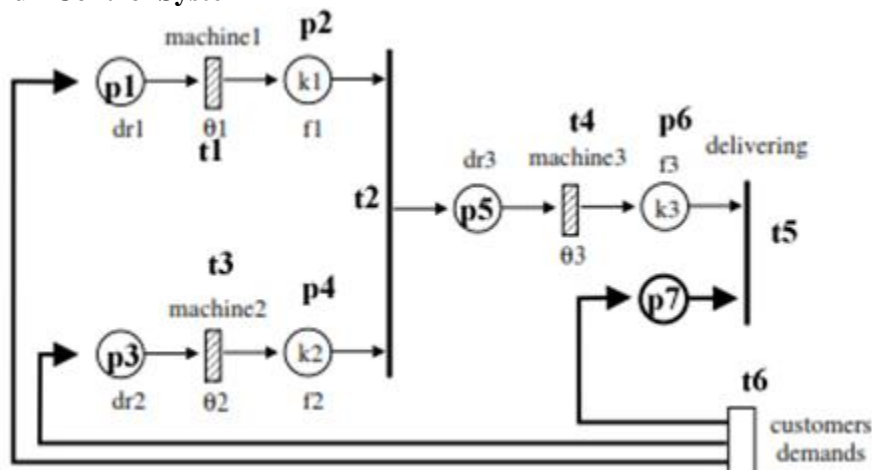


Figure 1: Production of Parts A and B (stages 1 and 2) and final assembly (stage 3) with a basic stock (pull) control system (BSCS)

$$A_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ + \\ 0 \end{bmatrix} \text{ Place}_1 = \{p_1\} \text{ For } r=1 \quad V_{11} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ + \\ 0 \end{bmatrix} \text{ PLACE}_{11} = \{p_1\}$$

Step 2: V_{11} has no \pm entry. Therefore p_1 does not form self loop with any transition t_i

Step 3,4,5,6;

V_{11} has + entry in 5TH row there neutralizing column are A_1, A_8

$$V_{12}^{(1)} = V_{11} \oplus A_7 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ + \\ 0 \end{bmatrix} \oplus \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ + \\ - \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \pm \\ 0 \end{bmatrix} \text{ PLACE}_{12}^{(1)} = \{p_1, p_7\},$$

$$V_{12}^{(2)} = V_{11} \oplus A_8 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ + \\ 0 \end{bmatrix} \oplus \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ - \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \pm \end{bmatrix}$$

$V_{12}^{(1)}$ has a + entry in 4th row. Therefore neutralizing column are A_6

$$V_{13}^{(1)} = V_{12}^{(1)} + A_6 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ + \\ \pm \\ 0 \end{bmatrix} \oplus \begin{bmatrix} 0 \\ 0 \\ + \\ - \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \pm \\ \pm \\ \pm \\ 0 \end{bmatrix} \text{ PLACE}_{13}^{(1)} = \{p_1, p_7, p_6\}$$

$V_{12}^{(2)}$ has + entry in 6th row. But there is no(-) sign in 6th row. Therefore no neutralizing column. Here $\text{PLACE}_{13}^{(1)} = \{p_1, p_7, p_6\}$ forms set of places which are both siphon and trap.

Example 3.3 Independent Kanban Control System .

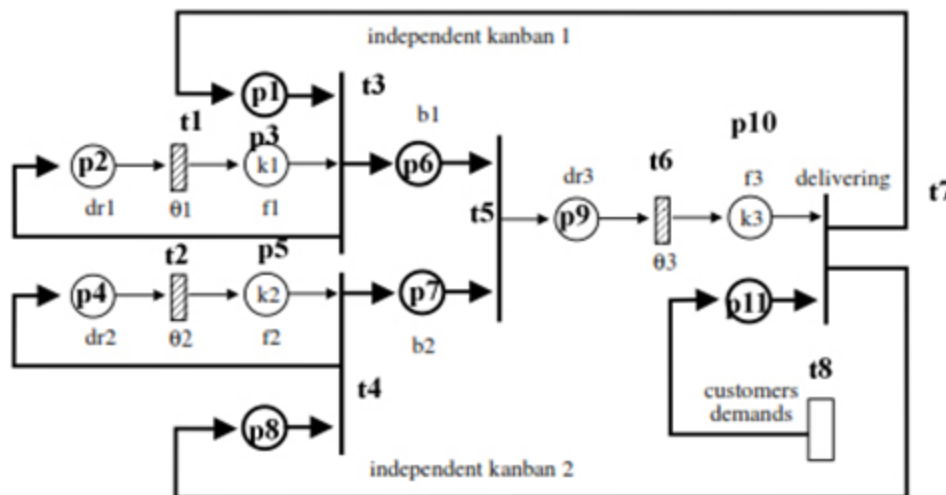


Figure 3: Independent Kanban Control System(IKCS)

The sign incidence matrix for the above marked graph is

$$B_3 = \begin{matrix} & \begin{matrix} p_1 & p_2 & p_3 & p_4 & p_5 & p_6 & p_7 & p_8 & p_9 & p_{10} & p_{11} \end{matrix} \\ \begin{matrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \\ t_6 \\ t_7 \\ t_8 \end{matrix} & \begin{bmatrix} 0 & - & + & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & - & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ - & + & - & 0 & + & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & + & 0 & + & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & - & - & 0 & 0 & + & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & - & + & 0 \\ + & 0 & 0 & 0 & 0 & 0 & 0 & + & 0 & - & - \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & + \end{bmatrix} \end{matrix}$$

Applying Algorithm 2.7

Step 1

$$A_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ + \\ 0 \end{bmatrix} \text{ PLACE}_1 = \{P_1\}$$

Step 2;

V_{11} has a + entry in 7th row the neutralizing columns are A_{10}, A_{11} .

$$\text{Step 3; Proceeding as in the algorithm. We get } V_{14}^{(2)} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ + \\ 0 \\ + \\ + \\ + \\ + \\ 0 \end{bmatrix} \text{ PLACE}_{14}^{(2)} = \{p_1, p_9, p_6\}$$

Since all the entries zero (or) \pm the set of places. $\{p_1, p_9, p_6\}$. From both siphon and trap. There are no other set of places in this marked graph which are both siphon and trap.

Example 3.4 Independent extended Kanban control system

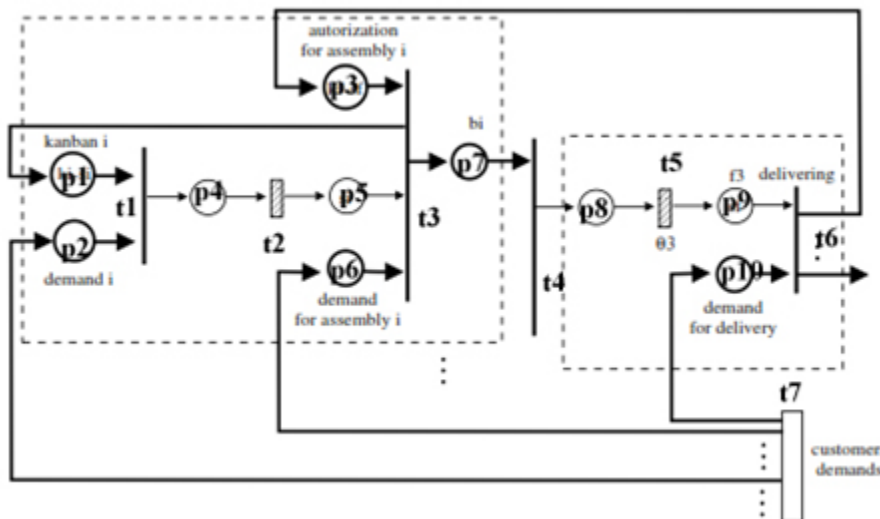


Figure 4: Independent extended Kanban control system(IEKCS)

The sign incidence matrix for the above marked graph is

$$B_4 = \begin{matrix} & \begin{matrix} p_1 & p_2 & p_3 & p_4 & p_5 & p_6 & p_7 & p_8 & p_9 & p_{10} \end{matrix} \\ \begin{matrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \\ t_6 \\ t_7 \end{matrix} & \begin{bmatrix} - & - & 0 & + & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & - & + & 0 & 0 & 0 & 0 & 0 \\ + & 0 & - & 0 & - & - & + & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & - & + & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & - & + & 0 \\ 0 & 0 & + & 0 & 0 & 0 & 0 & 0 & - & - \\ 0 & + & 0 & 0 & 0 & 0 & 0 & 0 & 0 & + \end{bmatrix} \end{matrix}$$

Step 1:

$$A_1 = \begin{bmatrix} 0 \\ + \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ PLACE}_1 = \{p_1\} \text{ For } r=1, V_{11} = \begin{bmatrix} 0 \\ + \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ PLACE}_{11} = \{p_1\}$$

Step2:

V_{11} has + entry in third zero . therefore the neutralizing column are A_3, A_5, A_6 Proceeding as in the algorithm we get

$$V_{13}^{(3)} = \begin{bmatrix} \pm \\ \pm \\ \pm \\ \pm \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ PLACE}_{13}^{(2)} = \{p_1, p_3, p_4\}$$

Here all the entries are either zero (or) \pm . Therefore the above set of places forms both siphon and traps. There are no other set of places in this marked graph which forms both siphon and traps.

CONCLUSION

From analyzing the above marked graphs we found the following

We summarize the results by giving the set of places which form siphon and trap in each of the seven marked graphs.

Name of the marked graph of fms	Set of places which form both siphon and trap
basic stock control system(BSCS)	Nil
Simultaneous Kanban Control System(SKCS)	$Z = \{p_1, p_7, p_6\}$
Independent Kanban control system(IKCS)	$Z = \{p_1, p_9, p_6\}$
Independent extended Kanban control system(IEKCS)	$Z = \{p_1, p_5, p_4\}$

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